APPENDIX: TECHNICALITIES OF Computational Fluid Dynamics (CFD) CALCULATIONS

Governing Equations for CFD simulations

In each geometric internal carotid artery aneurysm model, the CFD modeling solved the following Navier-Stokes equations for incompressible blood flow:

$$\frac{\partial V_i}{\partial x_i} = 0$$
$$\rho\left(\frac{\partial V_i}{\partial t} + V_j \frac{\partial V_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ji}}{\partial x_j} \qquad (1)$$

where ρ is the density, V_i is the velocity vector, p is the pressure, and τ_{ij} is the deviatoric stress tensor, related to the shear rate tensor e_{ij} by viscosity μ (in general, not a constant):

$$\tau_{ij} = 2\mu e_{ij} \quad , where \quad e_{ij} = \frac{1}{2}\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right) \qquad (2)$$

Defining a scalar shear rate value of $\dot{\gamma} = \sqrt{2e_{ij}e_{ij}}$ **,** the above relation becomes an algebraic equation in simple shear flow:

$$\tau = \mu\dot{\gamma} \qquad (3)$$

where τ is the shear stress, μ is the dynamic viscosity, and $\dot{\gamma}$ is the shear rate. Under the Newtonian fluid assumption, the viscosity of blood is constant, and a typical value of μ=3.5 centipoise (cP) is used. For non-Newtonian fluids, μ is dependent upon the shear rate $\dot{\gamma}$.

Two widely used non-Newtonian fluid models for blood, Casson and Herschel-Bulkley (H-B), were investigated in this study.[1]

The classic Casson model assumes a stress/strain rate relation:

$$\sqrt{\tau} = \sqrt{\tau_0} + \sqrt{\mu_\infty}\sqrt{\dot{\gamma}} \qquad (4)$$

where τ_0 is the yield stress and μ_∞ is the Newtonian viscosity. The existence of a yield stress means that blood needs a finite stress before it starts to flow, which has been observed by flow experiments. Thus, the apparent viscosity of the Casson model can be written as:

$$\mu = \left(\sqrt{\frac{\tau_0}{\dot{\gamma}}} + \sqrt{\mu_\infty}\right)^2 \qquad (5)$$

Because the above expression diverges when the strain-rate becomes zero, it is usually modified as follows:[2]

$$\mu = [\sqrt{\tau_0(\frac{1-e^{-m\dot{\gamma}}}{\dot{\gamma}})} + \sqrt{\mu_\infty}]^2 \qquad (6)$$

Typical values for blood are: τ_0 =0.035 $dyne/cm^2$, μ_∞ =0.035 $dynes/cm$, and m =100 s.

The H-B fluid model for blood assumes that the viscosity μ varies according to the following equations:[3]

$$\mu = k\dot{\gamma}^{n-1} + \frac{\tau_0}{\dot{\gamma}} \qquad (7)$$

Recommended experimental values for blood are k =8.9721 $cP \cdot s^{n-1}$, n =0.8601, and τ_0 =17.5 mPa [4]. This model, which provides a good fit over a range of shear rates, has been particularly popular in recent studies.[1, 3, 5-7] The density of blood is assumed constant, ρ =1056 kg/m^3, for both Newtonian and non-Newtonian simulations. Figure 2 in the manuscript shows viscosity plotted against shear rate for all rheology models evaluated in this study. These models give different stress-strain rate relations at low shear rates but exhibit similar viscosity behavior at the high shear rates. Both non-Newtonian models take into account the shear thinning (the viscosity increases with decreasing shear rate) as well as the yield stress behavior of blood.

Numerical Method

Finite volume tetrahedral meshes with wall prism meshes (for accurate boundary layer resolution) consisting of approximately 300,000 to 1 million elements were created for each internal carotid artery (ICA) intracranial aneurysm (IA) model using ANSYS ICEM CFD (ANSYS, Inc., Canonsburg, PA). The numerical solution of the incompressible Navier-Stokes equations under pulsatile flow conditions was obtained using the solver Star-CD (CD Adapco, Melville, NY). In all simulations, the same mean flow rate was used as the inlet boundary condition for the ICA (4.6 ml/s).[8] The shape of the pulsatile velocity waveform was obtained from transcranial Doppler ultrasound measurement on a normal subject, and its magnitude was scaled to the desired mean flow rate. Traction-free boundary conditions were implemented at the outlet. The mass flow rate through each outlet artery was proportional to the cube of its diameter based on the principle of optimal work.[9]

Three pulsatile cycles were simulated to ensure that numerical stability had been reached. All data presented in the manuscript are time-averages over the third pulsatile cycle of flow simulation. Shear rate $\dot{\gamma}$, Shear stress τ, and blood viscosity μ distributions at the lumen of the parent vessel were plotted for the three patient-specific aneurysms using all three rheology models. When evaluated at the luminal wall, shear stress is known as wall shear stress (WSS), which is the frictional force from flowing blood to which the endothelium is exposed. For ease of comparison of the results data, the shear rate, WSS, and viscosity were normalized by the local values calculated from the Newtonian model.

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